

San Diego City Schools

Institute for Learning

MATHEMATICS DEPARTMENT

K-12 Mathematics Framework

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Note: The content in this document was distributed as a draft released by San Diego City Schools on June 6, 2000.

San Diego City Schools K-12 Mathematics Framework

Introduction

The San Diego City Schools K-12 Mathematics Framework is a document designed to improve student achievement. The framework will serve as a guide for building and implementing a comprehensive curriculum that is coherent across grades, focused on high-quality mathematics and expects excellence in the teaching and learning of mathematics.

Built upon district standards,¹ the framework is organized into two categories:

1. Mathematical content – the conceptual strands of mathematics.
2. Mathematical processes – the tools and habits of mind people use when solving problems.

The 1996 NAEP results demonstrate that many students are able to compute, but cannot solve problems. The San Diego City Schools Mathematics Framework includes the breadth and depth of content students need to understand mathematics, so that they can compute and solve problems. Students who learn the mathematics put forth in the framework will be resourceful and flexible problem solvers who can effectively apply mathematical knowledge. They will be able to represent mathematical knowledge in a variety of ways to meet the demands of higher education and the world of work.

The San Diego City Schools Mathematics Framework establishes a system to link and align district mathematics standards, professional development, instructional practice, and curricular materials. The framework focuses and organizes mathematics content and instructional practice, leading the way to improved mathematics learning for all students in the San Diego City Schools.

¹The elements of the framework also are convergent with those of the *Mathematics Framework for California Public Schools*, the *Standards in Mathematics for California High School Graduates* and the *2000 NCTM Principles and Standards for School Mathematics*.

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Mathematical Content

1. Number Sense and Operations

Students develop number sense as they learn to think and reason flexibly, make sound numerical judgments and are able to judge what's numerically reasonable in various situations. When faced with a situation that calls for numerical calculations, students need to be able to choose the correct operations, decide on the numbers to use, do the necessary calculation, and then appropriately interpret the results. Development of a deep and fundamental understanding of numbers and operations continues throughout the grades.

2. Functions and Algebra

Algebra emphasizes relationships among quantities, including functions, ways of representing mathematical relationships and the analysis of change. Instructional programs designed to provide ongoing experiences with patterns lead to an understanding of function. Experiences with numbers and their properties lay the foundation for later work with symbols and algebraic expressions. Viewing functions and algebra as a strand in the curriculum at all grades builds the solid foundation of understanding and experience that prepares students for more complex work in algebra in the middle grades and high school.

3. Measurement and Geometry

Measurement and geometry provide students ways to interpret and understand their physical environment. They provide tools for the study of other topics in mathematics and science. Across the grades, students build an understanding of units of measure and learn to apply appropriate techniques, tools and formulas. They learn about geometric shapes and structures and how to analyze their characteristics and relationships. An important aspect of geometric thinking is spatial visualization. School experiences provide opportunities for students to build and manipulate mental representations of two- and three-dimensional objects. They work with concrete models, drawing and dynamic computer software. Visualization, spatial reasoning and geometric modeling are strategies that help students analyze and solve problems

4. Data Analysis, Statistics and Probability

Data analysis, statistics, and probability develop students' ability to understand, analyze and evaluate statistical information, an important skill for an informed citizen, employee, and consumer. School experiences provide students opportunities to formulate questions, collect, organize and display relevant data. They learn methods of analyzing data and ways of making inferences and drawing conclusions. Students learn basic concepts and applications of probability, with the emphasis on the way that probability and statistics are related.

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Mathematical Processes

1. Quantitative Literacy

Students who understand numbers and the number system make sense of them, use them when solving problems, and recognize reasonable results. They can break numbers apart (for example, they can see the number 24 and know that it is 2 tens and 4 ones and also two sets of 12). Quantitatively literate students use appropriate numbers as referents (such as 10, 2, 5), solve problems using the operations and their relationships and apply their understanding of the base-ten system.

2. Computational Fluency

Computational fluency is essential. Students with computational fluency have the ability to use a variety of efficient procedures to perform calculations that produce accurate results. They use mental calculations, estimation, and paper and pencil calculations using mathematically accurate algorithms. In addition, students use calculators appropriately. Calculators should be set aside with the instructional focus is on developing or practicing computational algorithms.

3. Problem Solving

Problem solving means that students can engage in tasks for which the solution is not known in advance. Problem solving is both a goal and a means of learning mathematics. Problem solving helps students learn to deal with unfamiliar situations and develop habits of persistence. It requires that students explore, make conjectures and question one another. Students who are adept at problem solving can analyze situations in mathematical terms. Throughout the grades, students develop and expand a more complex set of tools such as using diagrams, looking for patterns or working backwards, and they learn to monitor and adjust strategies as they solve a problem.

4. Using Representations

Representations enable students to organize, record, and communicate mathematical ideas to themselves and to others. Students who understand representations can select, apply, and translate them when solving problems. Students learn and use conventional forms of mathematical symbolism, and develop the ability to represent their own mathematical ideas in ways that make sense and clearly communicate them to others. Forms of representation such as diagrams, graphical displays, and symbolic expressions are acquired as powerful and useful tools for learning and doing mathematics and for communicating the results of problem solving, reasoning, and proving or verifying results.

5. Using Reason and Proof

School experiences in reasoning and proof build from early experiences using inductive arguments to later work with deductive argument. Throughout secondary school, students deepen their understanding and use of mathematical proofs. K-12 experiences with reasoning and proof help students develop the skills necessary to make and investigate mathematical conjectures, and evaluate mathematical and other arguments.

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6. Communicating

Communication is an essential part of mathematics and mathematics instruction. Through communication, students organize and consolidate their mathematical thinking. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others, they learn to be clear and convincing. Discussions about mathematical ideas help students learn to analyze and evaluate the mathematical thinking and strategies of others. Communicating clearly, whether orally or in written form, requires that students use the language of mathematics to express mathematical ideas precisely.

7. Making Connections

Focusing on the connections among mathematical ideas helps students understand that mathematics is a coherent whole, rather than a collection of isolated skills and arbitrary rules. Students who use connections recognize how ideas in different areas are related, and are able to use insights gained in one context to verify conjectures in another. Students who make connections see and experience the interplay among mathematical topics, between mathematics and other subjects, and between mathematics and their own interests.

References

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